1 Arithmetic Sequences

An **arithmetic sequence** is a sequence in which each term is formed from the preceding term by *adding* a constant (positive or negative).

Complete the following for the sequence $7, 10, 13, 16, ...$

- Each term is determined by adding ______ to the previous term.
- Calculate the differences: $t_2 - t_1 =$______ $t_3 - t_2 =$______ $t_4 - t_3 =$______

Notice that there is a **common difference** between successive terms.

The common difference in this example is ______.

2 Finding a Common Difference

To find a common difference in an arithmetic sequence, we can subtract any term from the term after it. For example:

- $t_2 - t_1 =$ common difference, or
- $t_5 - t_4 =$ common difference, etc.

common difference =____________

2.1 Example

Consider the sequence $19, 15, 11, 7, ...$

The **common difference** is ______.

2.2 Example

Consider the sequence $21, 28, 35, 42, ...$

The **common difference** is ______.

2.3 Example

For each of the following

- determine which sequences are arithmetic
- find the common difference (if applicable)

1. $2, 4, 6, 8, ...$
2. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots$
3. $-10, -4, 2, 8, ...$
4. $4, 8, 16, 32, ...$

Vocabulary

In arithmetic sequences, we use the following terminology:

The **first term** in an arithmetic sequence is represented by t_1, or a, and the **common difference** is represented by d.

2.4 Example

State the values of a and d in the following sequences:

1. $-8, 2, 12, 22, ...$
2. $15, 10, 5, 0, ...$

2.5 Investigate!

Consider the sequence $2, 12, 22, 32, 42, ...$

State the following:

$t_1 =$ $t_2 =$ $t_3 =$ $t_4 =$ $t_5 =$ $a =$ $d =$

Complete the following pattern which describes each term in the sequence in terms of the first term, a, and the common difference, d.

$t_1 = 2$	$t_1 = a$
$t_2 = 2 + 1(10) = 12$	$t_2 = a + (1)d$
$t_3 = 2 + 2(10) = 22$	$t_3 = a + 2d$
$t_4 =$	$t_4 =$
$t_5 =$	$t_5 =$
$t_{30} =$	$t_{30} =$
$t_n =$	$t_n =$

3 Determining a Formula for the General Term of an Arithmetic Sequence

The formula for the general term of an arithmetic sequence is

$t_n = t_1 + (n - 1)d$ or $t_n = a + (n - 1)d$

where,

t_n is the general term of the arithmetic sequence

$a = t_1$ is the first term

d is the common difference

n is the position of the term in the sequence

The general arithmetic sequence is $a, a + d, a + 2d, a + 3d, ...a + (n - 1)d$.

3.1 Example

Consider the arithmetic sequence $-6, -1, 4, 9, ...$

Determine the formula for the general term of the sequence.

Determine the value of the twelfth term of the sequence.

3.2 Example

Find the number of terms in the arithmetic sequence $3, -1, -5, ..., -117$.

4 Arithmetic Means

The terms placed between two non-consecutive terms of an arithmetic sequence are called **arithmetic means**. For example, in the sequence $5, 10, 15, 20$, the numbers 10 and 15 are arithmetic means between 5 and 20. In order to determine arithmetic means between two given terms, it is helpful to think of the two given terms as the **first** and **last** terms of a sequence.

4.1 Example

Place three arithmetic means between -4 and 8.

5 Solving Sequence Problems Where Both "a" and "d" are Unknown

Consider the sequence $x + 2, 3x - 1, 2x + 1$.

Determine the value of x such that $x + 2, 3x - 1$, and $2x + 1$ form an arithmetic sequence.

Determine the numerical value of the three terms.

The next example shows another way to solve the same type of problem.

5.1 Example

The third and eighth terms of an arithmetic sequence are 12 and -18, respectively.

Use arithmetic menas to determine the fifth term of the sequence.

State the first term, a, and the common difference, d, of the sequence.

Complete the following: $\frac{t_8 - t_3}{8-3} = \underline{\hspace{2cm}} =$

Write t_3 and t_8 in terms of a and d and prove that $\frac{t_8 - t_3}{8-3} = d$.

Suggest a formula for finding the common difference of a sequence if you are given the value of the pth term and the qth term.